

Introduction to Proofs Final Exam Review Website

1.
 1. Write the following set using set-builder notation $\{1, 3, 5, 7, \dots\}$.
 2. List the elements of the following set $\{x : x^2 - 4 = 0\}$.
 1. Written Solution
 2. Video Solution
2. Let $A = \{1, 2, 4, 5, 7\}$, $B = \{2, 5, 6, 7, 8\}$, $C = \{1, 5, 7, 8\}$. and $U = \{x \in \mathbb{N} : x \leq 10\}$. Then draw a venn-diagram for A, B and C putting each of the elements of U in the appropriate region of the venn-diagram.
 1. Written Solution
 2. Video Solution
3. Let $A = \{a, b, c\}$ and $B = \{b, c, d\}$. Then find
 1. $A \cap B$
 2. $(A - B) \times B$
 3. $\mathcal{P}(A)$
 4. $\mathcal{P}(A - B) \times \mathcal{P}(B - A)$.
 5. What is $|\mathcal{P}(A) \times B|$.
 1. Written Solution
 2. Video Solution
4. Let P and Q be the logical statements given by $P = p \vee (q \rightarrow r)$ and $Q = (p \vee q) \wedge r$. Then use a truth table to determine if P implies Q , Q implies P , $P \equiv Q$ or none of the above are true.
 1. Written Solution
 2. Video Solution

5. Write the following statement using symbolic logic. Then give the negation of the statement symbolically. Finally, give this negation as a sentence.

Theorem 1. *Everyone who is sane can do logic. None of your sons can do logic. (Lewis Carroll).*

1. Write symbolically
2. Write the symbolic negation.
3. Write the negation as a sentence.

1. Written Solution
2. Video Solution

6. Suppose that you have a bag full of 30 balls and all 30 are unique.

1. You reach in and grab 7 balls all at once. How many different outcomes for this will there be?
2. Suppose you reach in and grab one ball at a time lining them up and leaving them off to the side after each one until you have grabbed 7 balls. How many different such line-ups will there be?

1. Written Solution
2. Video Solution

7. Suppose that you have a standard deck of 52 cards and you deal out a 5 card hand with all cards dealt at the same time.

1. How many ways can you deal a flush? (All cards the same suit)
2. How many ways can you deal a straight? (All cards in a row ex 2,3,4,5,6)
3. How many ways can you deal a straight or a flush?

1. Written Solution
2. Video Solution

8. Write an algorithm that sorts a given set of numbers from least to greatest.

1. Written Solution

9. Recall that $a \equiv b \pmod{n}$ if and only if $n|(b - a)$. Provide a proof for the following theorem. (Direct proof is suggested).

Theorem 2. *Let $a, b \in \mathbb{Z}$ and $n \in \mathbb{N}$. If $a \equiv b \pmod{n}$, then $a^2 \equiv b^2 \pmod{n}$.*

1. Written Solution

2. Video Solution

10. Prove the following theorem. (Proof by contrapositive suggested).

Theorem 3. *Let $a \in \mathbb{Z}$. If $10 \nmid a$, then $2 \nmid a$ or $5 \nmid a$.*

1. Written Solution

2. Video Solution

11. Prove the following theorem. (Proof by contradiction suggested).

Theorem 4. *There are no integer solutions to the equation $14x + 6y = 1$.*

1. Written Solution

2. Video Solution

12. Prove or disprove the following Theorem.

Theorem 5. *Let A and B be sets. Then $\mathcal{P}(A) \cap \mathcal{P}(B) = \mathcal{P}(A \cap B)$.*

1. Written Solution

2. Video Solution

13. Prove or disprove the following theorem.

Theorem 6. *For every natural number n , the integer $n^2 + 17n + 17$ is prime.*

1. Written Solution

14. Prove or disprove the following theorem.

Theorem 7. $\{13x + 10y : x, y \in \mathbb{Z}\} = \mathbb{Z}$.

1. Written Solution
2. Video Solution

15. Prove the following Theorem.

Theorem 8. *Prove that for all $n \in \mathbb{N}$, $\sum_{i=1}^n 2^i = 2^{n+1} - 2$.*

1. Written Solution
2. Video Solution