

Practice Exam 2

1. Recall that $a \equiv b \pmod{n}$ if and only if $n|(b - a)$. Provide a proof for the following theorem. (Direct proof is suggested).

Theorem 1. *Let $a, b \in \mathbb{Z}$ and $n \in \mathbb{N}$. If $a \equiv b \pmod{n}$, then $a^2 \equiv b^2 \pmod{n}$.*

1. Written Solution
2. Video Solution

2. Prove the following theorem. (Proof by contrapositive suggested).

Theorem 2. *Let $a \in \mathbb{Z}$. If $10 \nmid a$, then $2 \nmid a$ or $5 \nmid a$.*

1. Written Solution
2. Video Solution

3. Prove the following theorem. (Proof by contradiction suggested).

Theorem 3. *There are no integer solutions to the equation $14x + 6y = 1$.*

1. Written Solution
2. Video Solution

4. Prove or disprove the following Theorem.

Theorem 4. *Let A and B be sets. Then $\mathcal{P}(A) \cap \mathcal{P}(B) = \mathcal{P}(A \cap B)$.*

1. Written Solution
2. Video Solution