Practice Exam 2

1. Suppose that you have a bag full of 30 balls.

(i) Suppose that all 30 are unique.

 (a) Suppose that each of the 30 balls are different. You reach in and grab 7 balls all at once. How many different outcomes are there?

 (b) Suppose you reach in and grab one ball at a time lining them up and leaving them off to the side after each one until you have grabbed 7 balls. How many different such line-ups are there?

(ii) Suppose that there are 7 red balls, 13 green balls and 10 blue balls with each ball indistinguishable from any ball of the same color.

 (a) Suppose that you reach in and grab 7 balls at once. How many distinct outcomes are there?

 (b) Suppose you reach in and grab 1 ball at a time and put them off to the side in a line-up until all the balls are lined up. How many such line-ups will there be?

(i) Written Solution

(ii) Video Solution

2. Suppose that you have a standard deck of 52 cards and you deal out a 5 card hand with all cards dealt at the same time.

(i) How many ways can you deal a flush? (All cards the same suit)

(ii) How many ways can you deal a straight? (All cards in a row ex 2,3,4,5,6)

(iii) How many ways can you deal a straight or a flush?

(i) Written Solution

(ii) Video Solution

3. (i) Write out Pascal's triangle to row 6.

(ii) In the expansion of $(x + y)^5$ what is the coefficient in front of x^3y^2?

(iii) Let A be a set with $|A| = 6$, how many subsets of cardinality 4 does A have?

(i) Pascal's Triangle is,

$$
\begin{bmatrix}
1 \\
1 & 1 \\
1 & 2 & 1 \\
1 & 3 & 3 & 1 \\
1 & 4 & 6 & 4 & 1 \\
1 & 5 & 10 & 10 & 5 & 1 \\
1 & 6 & 15 & 20 & 15 & 6 & 1
\end{bmatrix}
$$

More on this here.

(ii) The coefficient in front of x^3y^2 is $\binom{5}{3} = 10$.

(iii) The number of subsets with cardinality 4 are $\binom{6}{4} = 15$.

4. Suppose that you flip a coin, roll a 6-sided die and pick a card out a deck. What is the probability you flip a heads, roll a 2 and pick an Ace?

Probability is $\frac{1*2*4}{2*6*52}$.

5. Suppose that you flip a coin 8 times. What is the probability you get a heads on the first flip, or you get exactly 4 heads?

Probability of heads on first flip is $\dfrac{1}{2}$.

Probability exactly four heads is $\dfrac{\binom{8}{4}}{2^8}$.

Probability of both is $\dfrac{\binom{7}{3}}{2^8}$.

Probability of at least one is $\dfrac{1}{2} + \dfrac{\binom{8}{4}}{2^8} - \dfrac{\binom{7}{3}}{2^8}$.