Final Exam Review

1. Let $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ defined by $f(m, n) = (5m + 4n, 4m + 3n)$. Is f a function? Is it one-to-one? Is it onto?

1. Written Solution
2. Video Solution

2. Prove that matrix multiplication is a noncommutative operation on the set of all 2×2 matrices with real entries.

1. Written Solution
2. Video Solution

3. Let F be the set of all mappings from $\mathbb{R} \to \mathbb{R}$. Define $+$ on F as, for all $f, g \in F$, $(f + g)(x) = f(x) + g(x)$. Show that, with this operation, F is a group.

1. Written Solution
2. Video Solution

4. Let F be the set of all mappings from $\mathbb{R} \to \mathbb{R}$. Show that f is not a group with respect to composition of functions.

1. Written Solution
2. Video Solution

5. Show that $n\mathbb{Z}$ is a subgroup of $\mathbb{Z}$ (using the normal $+$) for all $n \in \mathbb{Z}$.

1. Written Solution
2. Video Solution

6. Define a relation $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x - y \in \mathbb{Z}\}$. Prove that R is an equivalence relation. Find the partition of $\mathbb{R}$ created by the equivalence classes.

1. Written Solution

2. Video Solution

7. Show that congruence modulo n is a congruence relation on $\mathbb{Z}$ for all $n \in \mathbb{Z}$ and for both addition and multiplication.

1. Written Solution
2. Video Solution

8. Prove that $A \times B$ is commutative if and only if A and B are commutative.

9. Let $G = S_3$.

1. Find all the subgroups of G.
2. Determine the order of every element of G.
3. Determine the left cosets of each subgroup of G.
4. Determine the right cosets of each subgroup of G.
5. Which of the subgroups are normal subgroups?
6. For all normal subgroups, N, find G/N.
7. Give the subgroup lattice of G.

1. Written Solution
2. Video Solution

10. Let G, H and K be groups. If $\alpha : G \to H$ and $\beta : H \to K$ are isomorphisms, show that $\beta \circ \alpha : G \to K$ is an isomorphism.

11. Let G be a cyclic group of order n. Show that G is isomorphic to $\mathbb{Z}_n$.

12. Let $\theta : G \to H$ be a homomorphism from group G to group H. Let A be an Abelian subgroup of G. Show that $\theta(A)$ is an abelian subroup of H.

13. Describe the group $\mathbb{Z}_6/\langle\{2\}\rangle$.

14. Find all homomorphic images of $\mathbb{Z}_8$.

15. Show that the set of 2×2 matrices with real entries with the usual matrix addition and multiplication, $M(2, \mathbb{R})$, is a ring with unity.

16. Let p be prime, show that $\mathbb{Z}_p$ is an integral domain.

17. Find the smallest subfield of $\mathbb{R}$ containing 1.

18. Let R be a ring with characteristic n. Show that n divides $|R|$.

19. Let R be a commutative ring. Show that $R[x]$ is also a commutative ring.

20. Let p be prime. Then every element of $\mathbb{Z}_p$ is a root of $x^p - x$ in $\mathbb{Z}_p[x]$.