

Practice Exam 3

Name:_____

Make sure to neatly and clearly show all work for each problem.

1. Let $f(x) = x + 1$ and $g(x) = 3x + 1$ in $C[0, 1]$ using the inner product

$$\langle f, g \rangle = \int_0^1 f(x)g(x)dx$$

1. Find $\|f\|$.
2. Find $d(f, g)$.
3. Find $f \cdot g$.
4. Find the angle between f and g . (Writing your answer in terms of an arccos is acceptable).

1. Written Solution

2. Video Solution

2. Apply the Gram-Schmidt orthonormalization process to transform the following set of vectors into an orthonormal basis for \mathbb{R}^3 .

$$S = \{(1, 1, 2), (1, 1, 1), (0, 1, 1)\}$$

1. Written Solution

2. Video Solution

3. Find the orthogonal complement S^\perp of the set

$$S = \text{span}((1, 2, 1, 0), (1, 2, 2, 2))$$

1. Written Solution

4. Let $T(v) : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be given by $T(\mathbf{v}) = A\mathbf{v}$ where

$$A = \begin{bmatrix} 2 & 1 & 4 \\ 2 & -1 & 2 \\ 4 & 0 & 6 \end{bmatrix}$$

1. Find $\ker(T)$.
2. Find $\text{range}(T)$.
3. What is the rank of T ?
4. What is the nullity of T ?
5. Is T one-to-one, onto, an isomorphism or none of the above?
 1. Written Solution
 2. Video Solution
5. Let $T_1(x, y) = (2x + y, 5x + 3y)$ and $T_2(x, y) = (x + 2y, 3x + 4y)$.
 1. Find the standard matrix of T_1 .
 2. Find the standard matrix of T_2 .
 3. Find the standard matrix of $T_1 \circ T_2$.
 4. Find the standard matrix for T_1^{-1} ?
 1. Written Solution
6. Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be given by $T(x, y, z) = (2x - 2y + 3z, 3y - 2z, -y + 2z)$.
 1. Find the standard matrix, A , of T .
 2. Find a basis, B , of \mathbb{R}^3 such that the matrix of T with respect to B is diagonal.
 3. What is the transition matrix, P , from the standard basis to B .
 4. What is the matrix of T with respect to B ?
 1. Written Solution
 2. Video Solution