

## Final Exam Review

1.
  1. Write the following set using set-builder notation  $\{1, 3, 5, 7, \dots\}$ .
  2. List the elements of the following set  $\{x : x^2 - 4 = 0\}$ .
1. Written Solution at  
<https://dralb.com/2019/02/05/sets-finding-elements-or-defining-rules/>
2. Let  $A = \{1, 2, 4, 5, 7\}$ ,  $B = \{2, 5, 6, 7, 8\}$ ,  $C = \{1, 5, 7, 8\}$ . and  $U = \{x \in \mathbb{N} : x \leq 10\}$ . Then draw a venn-diagram for  $A, B$  and  $C$  putting each of the elements of  $U$  in the appropriate region of the venn-diagram.
1. Written solution at  
<https://dralb.com/2019/02/05/venn-diagrams/>
3. Let  $A = \{a, b, c\}$  and  $B = \{b, c, d\}$ . Then find
  1.  $A \cap B$
  2.  $(A - B) \times B$
  3.  $\mathcal{P}(A)$
  4.  $\mathcal{P}(A - B) \times \mathcal{P}(B - A)$ .
  5. What is  $|\mathcal{P}(A) \times B|$ .
1. Written solution at  
<https://dralb.com/2019/02/05/cartesian-product-and-power-sets/>
4. Let  $P$  and  $Q$  be the logical statements given by  $P = p \vee (q \rightarrow r)$  and  $Q = (p \vee q) \wedge r$ . Then use a truth table to determine if  $P$  implies  $Q$ ,  $Q$  implies  $P$ ,  $P \equiv Q$  or none of the above are true.
1. Written solution at  
<https://dralb.com/2019/02/05/truth-tables/>
2. Video solution at  
<https://youtu.be/LdU6wx1LU78>

5. Write the following statement using symbolic logic. Then give the negation of the statement symbolically. Finally, give this negation as a sentence.

**Statement 1.** *Everyone who is sane can do logic. None of your sons can do logic. (Lewis Carroll).*

1. Write symbolically
2. Write the symbolic negation.
3. Write the negation as a sentence.

1. Written solution at  
<https://dralb.com/2019/02/05/translating-to-symbolic-logic-and-back/>
2. Video solution at  
<https://youtu.be/YFZ0Bb0hf08>

6. Suppose that you have a bag full of 30 balls and all 30 are unique.

1. You reach in and grab 7 balls all at once. How many different outcomes for this will there be?
2. Suppose you reach in and grab one ball at a time lining them up and leaving them off to the side after each one until you have grabbed 7 balls. How many different such line-ups will there be?

1. Written solution at  
<https://dralb.com/2019/02/25/counting-balls/>

7. Suppose that you have a standard deck of 52 cards and you deal out a 5 card hand with all cards dealt at the same time.

1. How many ways can you deal a flush? (All cards the same suit)
2. How many ways can you deal a straight? (All cards in a row ex 2,3,4,5,6)
3. How many ways can you deal a straight or a flush?

1. Written solution at  
<https://dralb.com/2019/02/25/how-many-ways-can-you-get-a-straight-or-a-flush>

8. Recall that  $a \equiv b \pmod n$  if and only if  $n|(b - a)$ . Provide a proof for the following theorem. (Direct proof is suggested).

**Statement 2.** *Let  $a, b \in \mathbb{Z}$  and  $n \in \mathbb{N}$ . If  $a \equiv b \pmod n$ , then  $a^2 \equiv b^2 \pmod n$ .*

1. Written solution at  
<https://dralb.com/2019/02/26/direct-proof-a2b2mod-n/>
2. Video solution at  
<https://youtu.be/UDMsDaSr38Y>

9. Prove the following theorem. (Proof by contrapositive suggested).

**Statement 3.** *Let  $a \in \mathbb{Z}$ . If  $10 \nmid a$ , then  $2 \nmid a$  or  $5 \nmid a$ .*

1. Written solution at  
<https://dralb.com/2019/02/26/proof-by-contrapositive/>
2. Video solution at  
<https://youtu.be/3Magk02wGQg>

10. Prove the following theorem. (Proof by contradiction suggested).

**Statement 4.** *There are no integer solutions to the equation  $14x + 6y = 1$ .*

1. Written solution at  
<https://dralb.com/2019/02/26/proof-by-contradiction-there-are-no-integer-sol>
2. Video solution at  
<https://youtu.be/p77DmoRbA0g>

11. Prove or disprove the following Theorem.

**Statement 5.** *Let  $A$  and  $B$  be sets. Then  $\mathcal{P}(A) \cap \mathcal{P}(B) = \mathcal{P}(A \cap B)$ .*

1. Written solution at  
<https://dralb.com/2019/03/31/set-equality-proof/>

12. Prove or disprove the following theorem.

**Statement 6.** For every natural number  $n$ , the integer  $n^2 + 17n + 17$  is prime.

1. Written solution at  
<https://dralb.com/2019/03/31/disproving-a-theorem/>

**13.** Prove or disprove the following theorem.

**Statement 7.**  $\{13x + 10y : x, y \in \mathbb{Z}\} = \mathbb{Z}$ .

1. Written solution at  
<https://dralb.com/2019/03/31/integers-written-as-13x10y/>
2. Video solution at  
<https://youtu.be/k6RkxukPvVQ>

**14.** Prove the following Theorem.

**Statement 8.** Prove that for all  $n \in \mathbb{N}$ ,  $\sum_{i=1}^n 2^i = 2^{n+1} - 2$ .

1. Written solution at  
<https://dralb.com/2019/03/31/induction-proof/>
2. Video solution at  
<https://youtu.be/wL1QYvQ52EI>

**15.** Define a relation  $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x - y \in \mathbb{Z}\}$ . Prove that  $R$  is an equivalence relation. Find the partition of  $\mathbb{R}$  created by the equivalence classes.

1. Written solution at  
<https://dralb.com/2019/04/17/equivalence-relations-and-partitions/>
2. Video solution at

**16.** Let  $f : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z} \times \mathbb{Z}$  defined by  $f(m, n) = (5m + 4n, 4m + 3n)$ . Is  $f$  a function? Is it one-to-one? Is it onto?

1. Written solution at  
<https://dralb.com/2019/04/19/bijective-mappings/>

2. Video solution at

**17.** Prove that  $\lim_{x \rightarrow 2} (3x - 5) = 1$ .

1. Written solution at

<https://dralb.com/2019/04/19/proving-limits/>

2. Video solution at