Practice Exam 1

Name:_____

Make sure to neatly and clearly show all work and mark your answers.

1. Find

$$\int_0^1 x^3(x^4+2)^3 dx$$

1. Written Solution at
   `https://dralb.com/2019/02/02/integration-by-substitution/`
2. Video at `https://www.youtube.com/watch?time_continue=7&v=fY7BpscO93M`

2. Find the area bounded by the function $f(x) = 8x$, $g(x) = \frac{8}{x}$, and $h(x) = x^2$.

1. Written Solution at
   `https://dralb.com/2019/02/02/finding-area-using-integrals/`
2. Video at `https://www.youtube.com/watch?v=IVBx0Yku-sQ`

3. Suppose that R is the region bounded by $f(x) = e^{x^2}$, $y = 0$, $x = 0$ and $x = 1$. Furthermore, let this region be expanded to 3-dimensions by letting the cross sectional pieces be rectangles of height $f(x)$, and depth x. Find the volume of the described object.

1. Written Solution at
   `https://dralb.com/2019/02/02/volume-with-rectangular-cross-sections/`
2. Video at `https://www.youtube.com/watch?v=XZIHuvVhnSc`

4. Let R be the region bounded by $f(x) = x^2$ and $x = 1$. Find the volume of the object obtained by revolving R about the y-axis.

1. Written Solution at
   `https://dralb.com/2019/02/02/volume-using-shell-method/`
2. Video at `https://www.youtube.com/watch?v=3g6VUDCtGuY`

5. Let R be the region bounded by $f(x) = \ln(x^2)$, $y = 0$ and $x = e$. Find the volume of the object obtained by revolving this region about the y-axis.

1. Written Solution at
   https://dralb.com/2019/02/02/volume-using-washer-method/
2. Video at https://www.youtube.com/watch?v=iqqhr8XYd0g

6. Find the length of the curve of $f(x) = \frac{2}{3}(x+1)^{\frac{3}{2}}$ on the interval $[0, 3]$.

1. Written Solution at
   https://dralb.com/2019/02/02/arc-length-of-a-function/
2. Video at https://www.youtube.com/watch?v=CJSgCXzrtSk

7. Find the surface area of the object obtained by revolving

$$f(x) = \frac{1}{14}(e^{7x} + e^{-7x})$$

on $[-2, 2]$ around the x-axis.

1. Written Solution at
   https://dralb.com/2019/02/02/finding-surface-area-using-integrals/
2. Video at https://www.youtube.com/watch?v=9e3Tz0Ncmts

8. Find

$$\int \cos^2(x)\sin(x)dx$$

1. Written Solution at
   https://dralb.com/2019/02/16/integration-techniques-1-substitution/
2. Video at https://www.youtube.com/watch?v=ly7DaCCQ3H4

9. Find

$$\int \cos^2(x)\sin^2(x)dx$$

1. Written Solution at
   https://dralb.com/2019/02/17/integration-techniques-2-reduction-formulas/
2. Video at https://www.youtube.com/watch?v=oajwyQsnqQE

10. Find

$$\int_0^1 x^2 e^x dx$$

1. Written Solution at
   https://dralb.com/2019/02/18/integration-techniques-3-by-parts/
2. Video at https://www.youtube.com/watch?v=TGQKsY3HbVA

11. Find

$$\int \frac{x^2 - 18}{x^3 - 2x^2 + x} dx$$

1. Written Solution at
   https://dralb.com/2019/02/19/integration-techniques-4-partial-fractions/
2. Video at https://www.youtube.com/watch?v=SheNeORYOSw

12. Find

$$\int_0^8 \sqrt{64 - x^2} dx$$

1. Written Solution at
   https://dralb.com/2019/02/21/integration-techniques-5-trig-substitution/
2. Video at https://www.youtube.com/watch?v=xweAjgF_xVw

13. Find

$$\int_1^\infty e^{-x} dx$$

1. Written Solution at
   https://dralb.com/2019/02/23/integration-techniques-6-improper-integrals/
2. Video at https://www.youtube.com/watch?v=RTzb8GZkgnA

14. Find if the following sequence converges. If it does, provide the limit.

$$a_n = \frac{k^3 - 4k + \ln(k) - (2k)!}{k - k^{54} + k^k}$$

1. Written Solution at
   https://dralb.com/2019/03/26/limits-of-sequences/

2. No video.

15. Determine if the following series converges or diverges. If it converges, find what the sum is.

$$\sum_{k=1}^{\infty} \frac{2^{k+2}}{3^k}$$

1. Written Solution at
   https://dralb.com/2019/03/28/geometric-series/

2. No video.

16. Determine if the following series converges or diverges. If it converges, find what the sum is.

$$\sum_{k=1}^{\infty} \frac{3}{3k-2} - \frac{3}{3k+1}$$

1. Written Solution at
   https://dralb.com/2019/03/30/telescoping-series/

2. No video.

17. Determine if the series converges or diverges.

$$\sum \frac{2k^2+1}{\sqrt{k^3+2}}$$

1. Written Solution at
   https://dralb.com/2019/03/30/divergence-test/

2. No video.

18. Determine if the series converges or diverges.

$$\sum \left(\frac{k}{2k+3}\right)^{2k}$$

1. Written Solution at
   https://dralb.com/2019/03/30/root-test/
2. No video.

19. Determine if the series converges absolutely, conditionally, or diverges.

$$\sum \frac{(-1)^k}{k \ln(k)}$$

1. Written Solution at
   https://dralb.com/2019/03/30/alternating-and-integral-test/
2. Video at https://www.youtube.com/watch?v=xGWGyArlQl8

20. Let $f(x) = x^{-1}$.

1. Find the Taylor Series for $f(x)$ centered at $x = 1$.
2. Determine the interval and radius of convergence of the Taylor Series.
3. Use the Taylor polynomial of degree 2 to approximate $\frac{1}{1.1}$.

1. Written Solution at
   https://dralb.com/2019/03/30/taylor-series/
2. Video at https://www.youtube.com/watch?v=wy3LqGj4yJo

21. Find the equation of the tangent line to the function given parametrically as

$$x(t) = t - \sin(t)$$
$$y(t) = 1 - \cos(t)$$

at the point corresponding to $t = \frac{\pi}{4}$.

1. Written Solution at
   https://dralb.com/2019/04/12/tangents-to-parametric-equations/
2. Video at https://youtu.be/bQQ0yMlloww

22. Give the area of the region enclosed by all the leaves of $3\sin(4\theta)$.

23. Find the slope of the tangent line $\frac{dy}{dx}$ to the polar curve given by $r = 3\sin(4\theta)$ for any θ.

1. Written Solution at
   `https://dralb.com/2019/04/13/leaves-of-polar-curves/`

2. Video at `https://youtu.be/4aY5NDS7vos`

24. Let $f(t) = (\sin(2t), 3\cos(4t), t)$. Find $F(t) = \int f(t)dt$ that satisfies the condition $F(0) = (2, 0, 3)$.

25. Let $f(t) = (\sin(2t), 3\cos(4t), t)$. Find $f'(t)$.

1. Written Solution at
   `https://dralb.com/2019/04/15/calculus-on-vector-functions/`